Problem Set #3

ECON 407: Mathematical Economics

(**due next class**)

1. (a) Give the dimensions of each of the following matrices. (b) Give their transposes and indicate their new dimensions.

$A=\left[\begin{matrix}6&7&9\\2&8&4\end{matrix}\right]$ $B=\left[\begin{matrix}12&9&2&6\\7&5&8&3\\9&1&0&4\end{matrix}\right]$ $C=\left[\begin{matrix}1\\2\\3\\4\end{matrix}\right]$ $D=[1]$

2. Let $A=I-X(X'X)^{-1}X'$, where $I$ is the identity matrix.

a) Does $A$ have to be a square matrix by the rules of conformability? Explain why or why not.

b) Does $(X^{'}X)$ have to be a square matrix? Explain why or why not.

c) Show that matrix $A$ is idempotent (i.e., $AA=A$). **Hint**: multiply $AA$ out and simplify using properties of the identity matrix and inverses.

3. Consider the situation of a mass layoff (i.e., a factory shuts down) where 1,200 people become unemployed and now begin a job search. In this case there are two states: employed (*E*) and unemployed (*U*) with an initial vector,

$$x'\_{0}=\left[\begin{matrix}E&U\end{matrix}\right]=\left[\begin{matrix}0&1200\end{matrix}\right]$$

Suppose that in any given period an unemployed person will find a job with probability 0.7 (thus, remaining unemployed with a probability of 0.3). Additionally, an employed person in any given period has a probability of 0.1 that they are fired (thus, having a 0.9 probability of keeping their job).

a) Setup the Markov transition matrix for this problem.

b) How many people will be unemployed after (i) 2 periods?; (ii) 3 periods?; (iii) *n* periods?

4. Let,

$$A=\left[\begin{matrix}1&0\\0&1\end{matrix}\right], B=\left[\begin{matrix}3&8&-1\\-5&0&6\end{matrix}\right], C=\left[\begin{matrix}1&2&3\\4&5&6\\7&8&9\end{matrix}\right]$$

Compute the following if it is defined:

a) $|A|$

b) $|B|$

c) $\left|C\right|-|A|$

d) Is matrix $C$ invertible? Explain why or why not.